$$\therefore \int_0^{\frac{1}{4}\pi} \log(\tan z)dz = -(1-\frac{1}{3^2}+\frac{1}{5^2}-\ldots.)=-\frac{\pi^3}{32},$$

by a well known summation.

A correct result was also received from Lon. C. Walker. The integration in finite terms required that the entire work should be done in finite terms as is done, for example, in Byerly's *Integral Colculus*, page 98, when

$$\int_0^{\frac{1}{2}\pi} \log\sin x dx$$

is found, and not the final result stated in finite terms.

The summation of the above series may be found by using the relation,

$$\frac{B_{2n}}{(2n)!}=\frac{2^{2n+2}}{\pi^{2n+1}}[1-\frac{1}{3^{2n}}+\frac{1}{5^{2n}}-\frac{1}{7^{2n}}+\ \ldots]$$

When $n=1$, $B_2=1$, one of Euler's numbers. See B. O. Peirce's *Table of Integrals*, page 90. ED.

124. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Show that the cardioides $r=a(1+\cos\theta)\ldots.(1)$, and $r=b(1-\cos\theta)\ \ldots(2)$, intersect at right angles.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and J. SCHEFFER, A. M., Hagerstown, Md.

The equation can be written $r=2a\cos^2\frac{1}{2}\theta$, $r=2b\sin^2\frac{1}{2}\theta$, or $r^{\frac{1}{2}}=(2a)^{\frac{1}{2}}\cos\frac{1}{2}\theta$, $r^{\frac{1}{2}}=(2b)^{\frac{1}{2}}\cos\frac{1}{2}(\pi-\theta)$.

$$\frac{dr}{rd\theta}=-\tan\tfrac{1}{2}\theta,\ \ \frac{dr}{rd\theta}=+\tan\tfrac{1}{2}(\pi-\theta).$$

At the point of intersection r and θ are the same for both.

The angle made by the perpendicular from the origin on the tangent is in the first case $\frac{1}{2}\theta$, in the second $\frac{1}{2}(\pi-\theta)$.

But $\frac{1}{2}\theta+\frac{1}{2}(\pi-\theta)=\frac{1}{2}\pi$.

$\therefore$ These perpendiculars are perpendicular to one another.

$\therefore$ The tangents are perpendicular and the cardioids intersect at right angles.

Solved substantially as above by *JOHN F. TRAVIS*, Fellow and Assistant in Mathematics in Ohio State University, Columbus, O.; *E. L. SHERWOOD, A. M.*, Professor of Mathematics, Beaver College, Beaver, Pa., and *L. C. WALKER*.

MECHANICS.

125. Proposed by T. U. TAYLOR, C. E., Professor of Civil Engineering, University of Texas, Austin, Tex.

(1) If a parabola is described on the vertical face of a reservoir wall, axis vertical and in the surface, and P (h, b) be any point on the curve, and B the foot of the perpendicular from P on the axis, find c. p. on area OBP.

(2) If A is point where horizontal through P cuts vertical axis (OY), find c. p. on area OAP.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

From statement of problem "axis vertical" should evidently read "axis horizontal."

Let $y^2=2px$ be the parabola. But $b^2=2ph$.

$\therefore y^2=b^2x/h$. Let p=pressure, m=moment, w=unit weight of water.

(1) $dp=wy(h-x)dy$, $dm=wy^2(h-x)dy$.

$$\therefore \bar{y}=\frac{\int_{-b}^{0} y^2(h-x)dy}{\int_{-b}^{0} y(h-x)dy}=\frac{\int_{-b}^{0} y^2(h-hy^2/b^2)dy}{\int_{-b}^{0} y(h-hy^2/b^2)dy}=-\tfrac{8}{15}b.$$

$$\bar{x}=\frac{\int_0^h y(h-x)^2dx}{\int_0^h y(h-x)dx}=\frac{\int_0^h \sqrt{x}\,(h-x)^2dx}{\int_0^h \sqrt{x}\,(h-x)dx}=\tfrac{4}{7}h.$$

(2) The depth of an elementary strip length x is y.

$\therefore dp=wxydy$, $dm=wxy^2dy$.

$$\therefore \bar{y}=\frac{\int_{-b}^{0} xy^2dy}{\int_{-b}^{0} xydy}=\frac{\int_{-b}^{0} y^4dy}{\int_{-b}^{0} y^3dy}=-\tfrac{4}{5}b.$$

$$\bar{x}=\frac{\int_0^h x^2(b-y)dx}{\int_0^h x(b-y)dx}=\frac{\int_0^h x^2\left(b-\frac{b\sqrt{x}}{\sqrt{h}}\right)dx}{\int_0^h x\left(b-\frac{b\sqrt{x}}{\sqrt{h}}\right)dx}=\frac{\int_0^h x^2(\sqrt{h}-\sqrt{x})dx}{\int_0^h x(\sqrt{h}-\sqrt{x})dx}=\tfrac{5}{21}h.$$

126. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, Gloucestershire, England.

AB is the horizontal base of a smooth cycloidal tube, vertex downward. A sphere is placed in the tube at A, and when it reaches the vertex another sphere of different mass is placed in the tube at B. When and where do they meet, and find their velocity immediately after collision, the spheres being partially elastic ?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let a be the radius of the generating circle, e the coefficient of restitution. m=mass of first sphere, m_1=mass of second sphere. It has been demonstrated that the first sphere will reach the vertex in the time, $t=\pi\sqrt{(a/g)}$, with a velocity, $v=2\sqrt{(ag)}$.